INTRINSIC CURVATURE OF SCHEMES

BY

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"Intrinsic Curvature For Schemes" a thesis prepared by Patrick M. Lank in partial fulfillment of the requirements for the degree, Masters of Science, has been approved and accepted by the following:

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DEDICATION

I dedicate this work to my mother, father, brother, and all of the wonderful people I have met along the way.

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ABSTRACT

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Dr. Alexandru Buium, Adviser

This thesis develops an algebraic analog of psuedo-Riemannian geometry for relative schemes whose cotangent sheaf is finite locally free. It is a generalization of the algebraic differential calculus proposed by Dr. Ernst Kunz in an unpublished manuscript to the non-affine case. These analogs include the psuedo-Riemannian metric, Levi-Civitá connection, curvature, and various existence theorems.

CONTENTS

1 Introduction

In this paper, a notion of the psuedo-Riemannian metric is presented for relative schemes whose contangent sheaf is finite locally free. For example, a smooth algebraic variety V of dimension n over a field k has this property that the cotangent sheaf $\Omega_{X/k}$ is locally free of rank n. More generally, nonsingular varieties have this property as well. The motivation may be found in the fact that there lacks a well-developed analog of a metric tensor in algebraic geometry. However, this does not mean that there has not been any efforts made to develop this lack of.

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